Estimating the dynamic and aerodynamic paramters of passively controlled high power rockets for flight simulaton

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February, 2009

Abstract

High power rockets are essentially large unguided model rockets that can fly to altitudes as high as 13 km and recover to earth by parachute. A rocket flight simulator can be used to predict the flight path of the rocket during the ascent. To do this the simulator requires certain input parameters. This paper summarizes some simple methods for estimating these parameters from very basic information such as the geometry of the rocket and the thrust curve. Methods for estimating the mass, centre of mass, moments of inertia, thrust damping, centre of pressure, drag force and normal force are given in this paper. The methodology described in this paper has been used in conjunction with a rocket flight simulator to predict the flight path of a high power rocket. Some results are given in the summary of the paper.

1 Introduction

Rocket flight simulators require certain input parameters that describe the properties of the rocket for the simulator. These include dynamic parameters such as the thrust, mass and moments of inertia of the rocket and also aerodynamic parameters such as the coefficients of the aerodynamic forces and the location of the centre of pressure.

The parameters can be obtained in a number of different ways, for instance in the case of the aerodynamic parameters these could be obtained through wind tunnel experiments on the rocket or on a scale model. Alternatively they can be found by using computational fluid dynamics to numerically integrate the viscous and pressure forces over the surface of the rocket.

While experimental and computational methods can provide accurate estimates of the parameters it is also useful to be able to estimate the parameters using simple equations derived from theoretical relationships using some simplifying assumptions. This approach provides a quick method for estimating parameters in applications where the most accurate estimates are not required.

This document collates and summarizes relevant equations for estimating the dynamic and aerodynamic input parameters from rocketry and engineering literature. In some cases equations are semi-empirical, that is they use relationships that have been fitted to data from experiments using rockets of a standard shape. In general, these equations are only applicable to rockets with a conventional shape. That is an axisymmetric rocket with a single cylindrical body, a conical, parabolic or ogive shaped nose cone and fin sets consisting of three or four trapezoidal fins. Changes is body diameter via conical transitions are allowed.

There are also a number of simplifying assumptions that are used in the estimation of the parameters and in the rocket simulators that use these parameters. These include the assumption that the rocket is an axisymmetric rigid body and that the angle of attack during the flight is always low. These and other assumptions will be discussed in more detail throughout the paper.

The methods presented here have been used to provide estimates of input parameters for a rocket flight simulator described in Box et al [2009], which also contains some validation data from a test flight.

2 The Dynamic Parameters

The dynamic parameters of the rocket that need to be estimated for flight simulations are the Thrust, mass, centre of mass, moments of inertia about the three principal axis and a thrust damping coefficient. Because the rocket is expelling mass from the motor nozzle all of these parameters will be time-varying until the motor burns out.

The thrust curve of the rocket motor can be measured on a test rig but in most cases commercially sold motors will be supplied with thrust-time data from the manufacturer. If the mass of the fuel is known then it is possible to estimate the mass curve for the motor by assuming that the mass burned at any point in time is proportional to the impulse of the motor up to that point (1).

$$\delta M_i = -\frac{M_f \displaystyle\int_0^i T\,dt}{\displaystyle\int_0^\infty T\,dt} \tag{1}$$

where M_f is the total mass of fuel, T is the thrust and δM_i is the change in the mass of fuel on board from ignition to time $t = i$. This calculation can be done numerically.

The following sections show how to estimate the centre of mass, moments of inertia and thrust damping coefficient for a given rocket mass M, which constitutes a single point on the mass curve. So to apply these calculations they must be repeated over the range of masses to build up time varying curves for each of these parameters also.

2.1 Centre of Mass and Moments of Inertia

For the purposes of these calculations it is assumed that the rocket is an axisymmetric body and the centre of mass of the rocket and it's component parts lies on the axis of symmetry, which is the roll axis of the rocket. We define the position of the centre of mass as a distance X_{cm} along the roll axis measured from the tip of the nose cone. The position of the centre of mass is defined using (2).

$$X_{cm} = \frac{1}{M}\int r\,dm \tag{2}$$

where r is the distance of an infinitesimal "slice" of rocket with mass dm along the roll axis and M is the total mass of the rocket.

Similarly the moment of inertia of the rocket about an axis of rotation is defined as

$$I = \int r^2 \, dm \tag{3}$$

where r is the distance of an infinitesimal volume with mass dm from the axis of rotation.

A simple approach to solving (2) and (3) is to divide the rocket up into component parts. The parts are then assumed to have simple shapes such as tubes, cones, blocks, cylinders etc. Small parts can be modelled as a point mass. Parts with complex shapes such as the packed parachute can approximated by a simpler shape, in this case a cylinder with uniform density.

The following subsections describe how to calculate the centre of mass and moments of inertia about the principle axes for some simple shapes and how to combine them together to estimate the total centre of mass and moments of inertia for the rocket.

2.1.1 Cone

The centre of mass of a solid cone is located on the axis of symmetry two thirds of the distance between the tip and the base.

$$X_{cm} = \frac{2l}{3} \tag{4}$$

where X_{cm} is distance from the cone's point to it's centre of mass and l is the height of the cone. The moments or inertia about the principal axes of a cone where the z axis is aligned with the cones axis of symmetry are given by

$$I_{xx} = I_{yy} = \frac{1}{10}Ml^2 + \frac{3}{20}Mr^2 \tag{5}$$

$$I_{zz} = \frac{3}{10}Mr^2 \tag{6}$$

where M is the mass of the cone and r is the radius of the base of the cone.

2.1.2 Tube

The centre of mass of a tube is located half way along the axis of symmetry.

$$X_{cm} = \frac{l}{2} \tag{7}$$

where X_{cm} is the distance of the centre of mass from one end of the tube and l is the length of the tube. The moments of inertia about the tube's principal axes, where the z axis is the axis of symmetry are

$$I_{xx} = I_{yy} = \frac{1}{12}M\left[3\left(r_1^2 + r_2^2\right) + l^2\right] \tag{8}$$

$$I_{zz} = \frac{1}{2}M\left(r_1^2 + r_2^2\right) \tag{9}$$

where M is the mass of the tube and r_1 and r_2 are the internal and external radii of the tube respectively

Block

The centre of mass of a block with sides of length X, Y and Z will be at the centre of the block ($X/2$, $Y/2$, $Z/2$).

The moments of inertia about a block's principal axis are given by

$$I_{xx} = \frac{1}{12}M(Y^2 + Z^2) \tag{10}$$

$$I_{yy} = \frac{1}{12}M(X^2 + Z^2) \tag{11}$$

$$I_{zz} = \frac{1}{12}M(X^2 + Y^2) \tag{12}$$

where M is the mass of the block.

Point Mass

Centre of mass of a point mass is it's location. A point mass has zero moment of inertia about it's own centre of mass.

Total Centre of Mass and Moments of Inertia

The location of the overall centre of mass of the rocket with reference to the nose cone tip is calculated using the following equation.

$$X_{cm(R)} = \frac{\sum_{P \in R} (X_P + X_{cm(P)}) M_P}{M_R} \tag{13}$$

where the subscript P denotes a part and R denotes the whole rocket. X_P is the distance from the nose tip to the most forward point of part P.

The overall moments of inertia are calculated using the parallel axis theorem.

$$I_P = I_{cm(P)} + M_p d^2 \tag{14}$$

$$I_R = \sum_{P \in R} I_P \tag{15}$$

where I_R is the moment of inertia of the rocket about a given axis through the rocket's centre of mass, I_P is the component of the rockets moment of inertia about the selected axis that is due to a given part P. $I_{cm(P)}$ is the moment of inertia about a parallel axis through the centre of mass of the part P, d is the distance between the two axes and M_P is the mass of the part.

Note that although a point mass has no intrinsic moment of inertia about it's own centre of mass it may still contribute to the moment of inertia of the rocket through the parallel axis theorem.

2.2 Thrust Damping

The expulsion of mass from the rocket nozzle has a further effect on the rocket's dynamics and this is to damp angular rotations of the rocket by generating a torque in the opposite direction to the rotation. This is caused by the lateral acceleration of the hot rocket gas as it travels down the motor tube [Duncan & Ensey, 1964].

The torque due to thrust damping is given by

$$\tau_{da} = -\dot{M}(l_{cn}^2 - l_{cc}^2)\omega \tag{16}$$

where $\dot{M}$ is the mass expulsion rate of the fuel, l_{cn} is the distance between the centre of mass of the rocket and the nozzle exit, l_{cc} is the distance between the centre of mass of the rocket and the centre of mass of the fuel and ω is the angular velocity of the rocket. The mass expulsion rate at time $t = x$ is $\frac{dM}{dt}\big|_{t=x}$ and can be estimated numerically from the mass time curve.

The term $\dot{M}(l_{cn}^2 - l_{cc}^2)$ is a dynamic parameter that must be passed to the simulator. We can define this term as the trust damping coefficient (C_{da}).

$$C_{da} = \dot{M}(l_{cn}^2 - l_{cc}^2) \tag{17}$$

$$\tau_{da} = -C_{da}\omega \tag{18}$$

However it should be noted that while it is conventional to call this a coefficient it is not dimensionless, it has the SI units $\frac{\mathrm{kg\,m^2}}{\mathrm{s}}$.

3 The Aerodynamic Parameters

3.1 Force Components

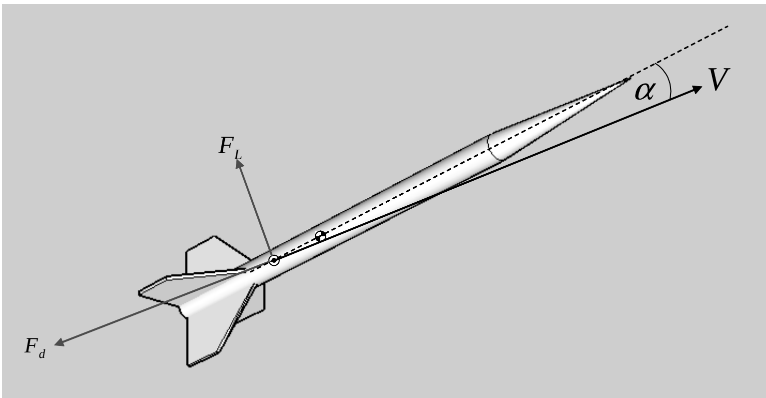

Figure 1: Showing the vectors of drag force F_D and lift force F_L relative to a rocket's apparent velocity vector V for flight with angle of attack α.

The aerodynamic force vector on a rocket in flight is typically considered as two orthogonal force vector components, however there are two conventions for how to orientate these components. Figure 1 shows the first convention. Here the components of aerodynamic force are the drag force F_D and the lift force F_L. These aerodynamic forces can be assumed to act on a single point in the rocket known as the centre of pressure. The vectors of the component forces

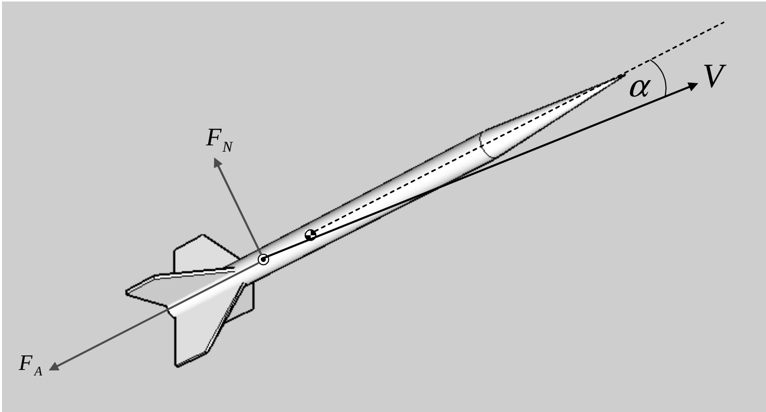

Figure 2: Showing the vectors of axial force F_A and normal force F_N relative to a rocket's roll axis.

are defined relative to the rocket's apparent velocity vector V. The apparent velocity is defined as velocity of the centre of pressure of the rocket relative to the atmosphere. The direction of the the drag force vector is exactly opposite to the direction of V.

The second convention is to orient the two orthogonal components relative to the rocket's roll axis as shown in Figure 2. Here the forces are referred to as the axial force F_A and the normal force F_N. As with drag and lift these forces act through the centre of pressure.

The angle between the vector defined by the rocket's roll axis and the apparent velocity vector V is known as the angle of attack α. The direction in which lift force or the normal force acts is orthogonal to the drag force or the axial force and in the plane defined by the roll axis vector and the apparent velocity vector. Also in both conventions the normal force, or the lift force goes to zero as the angle of attack α goes to zero.

The magnitude of any of these forces can be given by the following equation

$$F_i = \frac{1}{2}\rho V^2 A_r C_i \tag{19}$$

where the subscript i is either D,L,A,N for drag, lift, axial or normal respectively. ρ is the atmospheric density V is the rocket's apparent velocity A_r is the reference area, which is the cross-section area at the base of the nose cone and C_i is the relevant aerodynamic force coefficient. The challenge is to estimate these aerodynamic coefficients.

The aerodynamic forces are complex phenomena and there are no simple analytical solutions for the coefficients. In the incompressible flow regime the forces can be divided into pressure force and viscous force. Pressure force arises through the stagnation of fluid on the rocket forebody, fins, and any other protrusions, and also through a suction force created by a low pressure region at the base of the rocket where boundary layer separation occurs. Viscous force is due to skin friction between the rocket and the air. If the boundary layer at the rocket's surface is turbulent then the viscous forces will be significantly less that if it is laminar, hence the viscous force is highly dependant on Reynolds number.

As the rocket approaches the speed of sound a further force becomes relevant. At transsonic speeds shock waves form at the nose tip and at the leading edge of the fins. At sonic velocity these shocks are normal to the direction of motion of the rocket, and they become oblique (decreasing in half angle) as the rocket proceeds through super-sonic speeds. Momentum is transferred from the rocket to the surrounding air via these shock waves. This increased component of aerodynamic force is sometimes called *wave drag*.

At low speed (incompressible flow) the aerodynamic coefficients are functions of the angle of attack (α) and Reynolds number(Re). At higher speeds where the flow becomes compressible ($Ma \geq 0.4$) they are also functions of Mach number (Ma).

$$C_i = f(\alpha, Re, Ma) \tag{20}$$

3.2 Equations for estimating the Aerodynamic Coefficients

This section presents two separate methods from rocketry literature for estimating an aerodynamic coefficient using the rocket's geometry. The first is for estimating the normal force coefficient C_N and the second is for estimating the coefficient of drag C_D.

It might be considered more helpful to present two methods based on the same convention however this is not available in the literature. It is of course possible to combine these two methods to make estimates of either C_N and C_A or C_D and C_L. This however leaves the problem that two separate methods published by different authors are being used to estimate two components of what is essentially a single force, the aerodynamic force on the rocket.

Despite these misgivings, test data from real rocket flights has been shown to compare well with flight simulation data based on this method (see Section 4).

Figure 3 shows a schematic of an example rocket shape. On this figure key dimensions that are used to define the rocket's geometry are labelled. This figure will be referred to throughout the following sections.

3.2.1 Normal Force

Equations for estimating the normal force, and the location of the centre of pressure on a small rocket have been proposed by Barrowman [1998]. The Barrowman equations are derived using the following assumptions.

- The angle of attack of the rocket is low ($\alpha < 10°$)
- The effects of compressibility can be neglected i.e. $Ma < 0.4$
- Viscous forces are negligible
- Lift forces on the rocket body tube can be neglected
- The air flow over the rocket is smooth and does not change rapidly
- The rocket is thin compared to it's length
- The nose of the rocket comes smoothly to a point

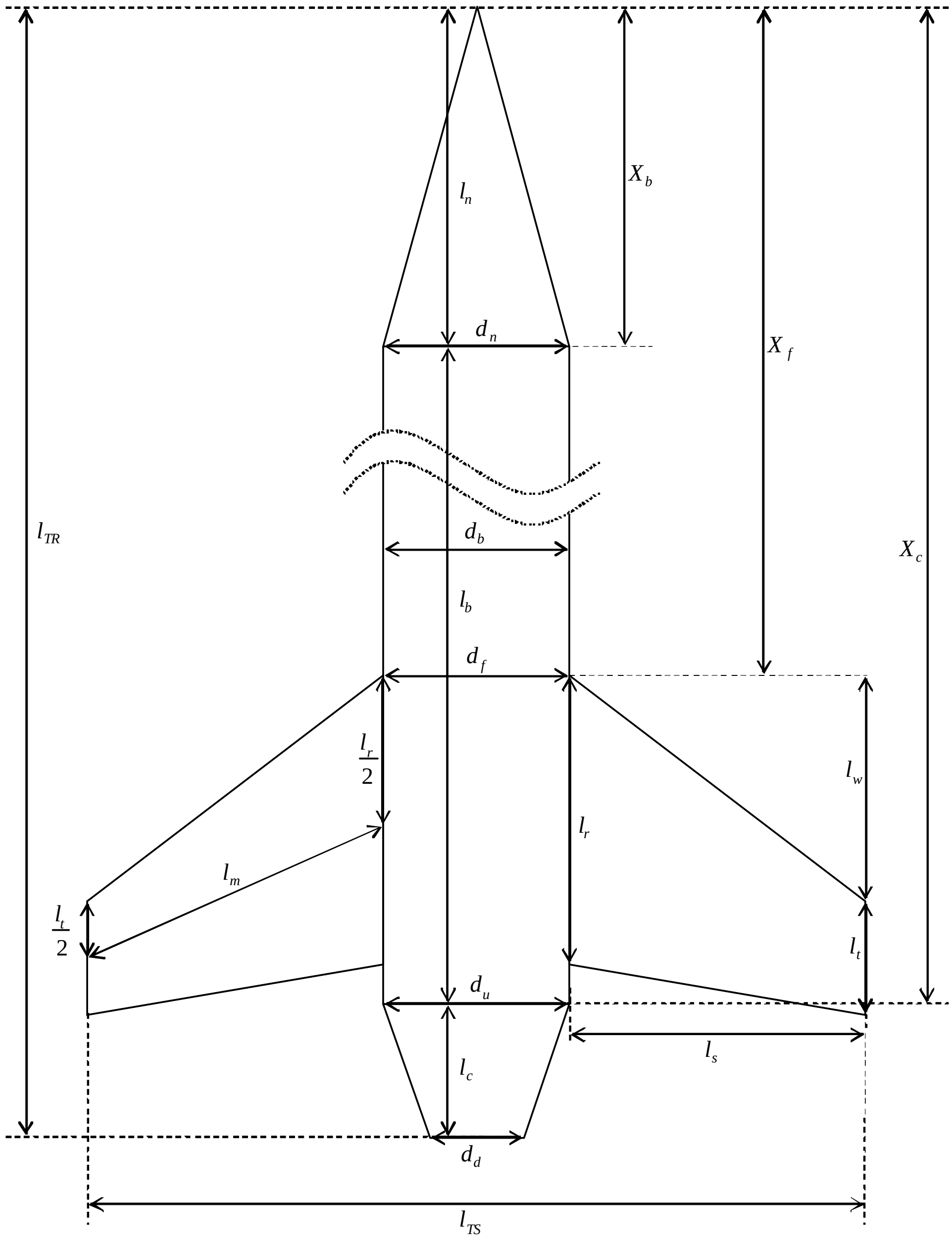

Figure 3: Schematic of a simple rocket with showing the principle dimensions

- The rocket is an axisymmetric rigid body
- The fins are thin flat plates

Here we will summarize the Barrowman method, a full derivation is given in Barrowman & Barrowman [1966]. From (19) the normal force on the rocket is given by (21)

$$F_N = \frac{1}{2}\rho V^2 A_r C_N \tag{21}$$

where A_r is the cross sectional area of the rocket at the base of the nosecone. The Barrowman method assumes incompressible flow and neglects viscous forces, therefore C_N is assumed to be a function of α only. Furthermore because a small α is assumed C_N can be expressed as a linear function of α (22).

$$C_N = C_{N\alpha}\alpha \tag{22}$$

where $C_{N\alpha}$ is the *slope* of the normal force coefficient, sometimes called the *stability derivative.*

$C_{N\alpha}$ is calculated as the sum of the individual stability derivatives for each of the components that makes up the rocket's shape. These components can be either a nosecone, a finset or a conical change in the diameter of the rocket's body tube. The body tube itself is neglected as a component because one of the key assumptions of the Barrowman method is that lift on the body tube is negligible. A rocket can consist of a nosecone and any number of finsets and conical changes in body diameter.

The total stability derivative is given by the sum of the individual stability derivatives for each of the parts of the rocket.

$$C_{N\alpha(R)} = \sum_{P\in R} C_{N\alpha(P)} \tag{23}$$

where P designates a rocket part and R designates the whole rocket.

The location of the rocket's centre of pressure is also calculated is terms of the centre of pressure of each of the individual rocket components. As with the centre of mass it is assumed that the centre of pressure lies on the roll axis of the rocket, the position of the centre of pressure is defined as the distance (X_{cp}) of the centre of pressure from the nose cone tip. X_{cp} is calculated using equation (24)

$$X_{cp(R)} = \frac{\displaystyle\sum_{P\in R} C_{N\alpha(P)} X_{cp(P)}}{C_{N\alpha(R)}} \tag{24}$$

where $X_{cp(P)}$ is the distance of the centre of pressure of component P from the tip of the rocket's nosecone.

The equations for calculating $C_{N\alpha}$ and X_{cp} for nosecones, finsets and conical changes in body diameter are given below.

Nose cone The stability derivative for a rocket's nosecone is 2 as long as the shape of the nosecone is either conical, ogive or parabolic.

$$C_{N\alpha(n)} = 2 \tag{25}$$

The location of the centre of pressure of the nose cone is dependent on the shape

$$\text{Conical: } X_{cp(n)} = \frac{2}{3} l_n \tag{26}$$
$$\text{Ogive: } X_{cp(n)} = 0.466 l_n \tag{27}$$
$$\text{Parabolic: } X_{cp(n)} = \frac{1}{2} l_n \tag{28}$$

where $X_{cp(n)}$ is the distance of the centre of pressure from the tip of the nose cone and l_n is the length of the nose cone.

Conical change in body diameter The stability derivative $C_{N\alpha}$ and the location of the centre of pressure X_{cp} for a conical change in body diameter are given by the following equations.

$$C_{N\alpha(c)} = 2\left[\left(\frac{d_d}{d_n}\right)^2 - \left(\frac{d_u}{d_n}\right)^2\right] \tag{29}$$

$$X_{cp(c)} = X_c + \frac{l_c}{3}\left[1 + \frac{1 - \frac{d_u}{d_d}}{1 - \left(\frac{d_u}{d_d}\right)^2}\right] \tag{30}$$

where d_n is the diameter of the base of the nose cone (reference diameter), d_u is the upstream diameter of the conical change, d_d is the downstream diameter of the conical change, l_c is the length of the conical change and X_c is the distance between the tip of the nose cone and the most upstream point on the conical change in body diameter (figure 3).

Equation (29) will produce a negative result if the upstream diameter is larger than the downstream diameter. This is correct as the pressure force in this case is a suction force acting towards the direction of flow.

Fins The following equations are for the entire fin set, and are valid for configurations with 3 or 4 trapezoidal fins.

The stability derivative for a trapezoidal finset is given by the following equation

$$C_{N\alpha(f)} = K_{fb} \frac{4n\left(\frac{l_s}{d_n}\right)^2}{1 + \sqrt{1 + \left(\frac{2l_m}{l_r + l_t}\right)^2}} \tag{31}$$

where n is the number of fins, l_s is the fin span, l_m is the fin mid-chord, l_r is the fin root-chord and l_t is the fin tip-chord (figure 3). K_{fb} is a coefficient that takes into account an increase in the normal force due to interference effects between the fin and the body.

$$K_{fb} = 1 + \frac{\frac{d_f}{2}}{(l_s + \frac{d_f}{2})} \tag{32}$$

where d_f is the diameter of the body tube at the fin's location. The location of the centre of pressure for a trapezoidal finset is given by

$$X_{cp} = X_f + \frac{l_m(l_r + 2l_t)}{3(l_r + l_t)} + \frac{1}{6}\left[l_r + l_t - \frac{l_r l_t}{l_r + l_t}\right] \tag{33}$$

where X_f is the length between the nose cone tip and the point where the fin leading edge meets the body tube.

Expanding the Valid Range of the Barrowman Equations As mentioned in the above section the Barrowman equations are subject to some quite restrictive assumptions. At least two of these assumptions may be broken in high powered rocket flight. The first of these is the assumption of incompressibility as high power rockets are capable of exceeding the speed of sound.

The second assumption is that the lift forces on the rocket body are negligible because the angle of attack α is below 10°. This assumption may be reasonable for most rockets but experiments have suggested that for rockets with particularly long slender bodies, body lift may not be negligible [Dahlquist, 1998].

Correcting for the effects of compressible flow is dealt with in section 3.2.3. Below we present a method for estimating the lift on the rocket body.

Rocket body lift correction Rocket body lift and it's associated effects on the position of a rockets' centre of pressure are neglected in the Barrowman equations. Galejs [1999] proposed the following extension to the Barrowman equations to take body lift into account.

The coefficient of normal force due to body lift is given by equation (34).

$$C_{N(L)} = K\frac{A_p}{A_r}\alpha^2 \tag{34}$$

where K is a constant between 1.0 and 1.5, A_p is the planform area of the rocket (excluding the fins) and A_r is the reference area for the rocket (cross-sectional area at the base of the nose cone).

$C_{N(L)}$ is not a linear function of α as with the assumption of the Barrowman method. Galejs defines

$$C_{N\alpha^2} = K\frac{A_p}{A_r}\alpha \tag{35}$$

which is not the slope of $C_{N(L)}$ but simply $\frac{C_{N(L)}}{\alpha}$. $C_{N\alpha^2}$ can be added to $C_{N\alpha}$ in equation (23). But this makes $C_{N\alpha}$ a function of α and hence the whole rocket C_N is non-linear.

The centre of pressure for the body lift force is taken to be the centre of the planform area, The equations used to determine the centre of the planform area for the various rocket body components are given below.

$$\text{Conical Nose: } X_{cp(n)} = \frac{2}{3}l_n \tag{36}$$

$$\text{Ogive Nose: } X_{cp(n)} = \frac{5}{8}l_n \tag{37}$$

$$\text{Parabolic Nose: } X_{cp(n)} = \frac{3}{5}l_n \tag{38}$$

$$\text{Body section: } X_{cp(n)} = X_b + \frac{1}{2}l_b \tag{39}$$

$$\text{Conical transition: } X_{cp(n)} = X_c + \frac{l}{3}\left[\frac{(d_u + 2d_d)}{(d_u + d_d)}\right] \tag{40}$$

For each component $C_{N\alpha^2}$ and X_{cp} are included in the Barrowman equations (23) and (24).

In the Galejs [1999] article results of this modelling were compared with experimental data for variation in CP with angle of attack from Dahlquist [1998]. The comparison shows a good agreement between experimental and modelled data for values $0° < \alpha < 15°$, using $K = 1$.

3.2.2 Drag Force

Equations for estimating the coefficient of drag force on a rocket are provided by Mandell et al [1973]. First we present how to calculate the drag force at zero angle of attack, then we show how so extend this to small angles of attack using semi-empirical equations.

Zero angle of attack drag force The coefficient of zero angle of attack drag force ($C_{D(0)}$) on a rocket can be estimated using the United States Air force Stability and Control Datcom Method [Mandell et al, 1973]. This method, like the Barrowman equations, uses basic information about the rocket geometry and assumes incompressible flow. to estimate the drag forces. The main equations of the datcom method are summarised below; derivations of the equations are given in Mandell et al [1973].

Body Drag The drag on the rocket forebody is estimated using equation (41)

$$C_{D(fb)} = \left[1 + \frac{60}{(l_{TR}/d_b)^3} + 0.0025\frac{l_b}{d_b}\right]\left[2.7\frac{l_n}{d_b} + 4\frac{l_b}{d_b} + 2\left(1 - \frac{d_d}{d_b}\right)\frac{l_c}{d_b}\right]C_{f(fb)} \tag{41}$$

where l_{TR} is the total length of the rocket body, l_c is the length of a boat tail (if present) - a boat tail is a conical reduction in the body diameter at the base of the rocket (figure 3). d_b is the maximum body diameter and d_d is the diameter of the rocket base. $C_{f(fb)}$ is the coefficient of viscous friction on the rocket forebody (defined later in (45))

Base Drag The base drag on the rocket is the drag due to the low pressure region at the base of the rocket that is caused by boundary layer separation. This drag is estimated using equation (42)

$$C_{D(b)} = 0.029\frac{\left(\frac{d_d}{d_b}\right)^3}{\sqrt{C_{D(fb)}}} \tag{42}$$

Fin drag The fin drag on the rocket at zero angle of attack is given by equation (43).

$$C_{D(f)} = 2C_{f(f)}\left(1 + 2\frac{T_f}{l_m}\right)\frac{4nA_{fp}}{\pi d_f^2} \tag{43}$$

where $C_{f(f)}$ is the coefficient of viscous friction on the fins (defined later in (45)), T_f is the fin thickness, n is the number of fins and d_f is the diameter of the body tube at the fin root.

A_{fp} is defined as the fin planform area. The planform area of the exposed part of a trapezoidal fin is given by $A_{fe} = \frac{1}{2}(l_r + l_t)l_s$. This is known as the *exposed area.* For the full planform area it is usual to assume that the fin extends to the centre line of the rocket body i.e. $A_{fp} = A_{fe} + \frac{1}{2}d_f l_r$.

Interference Drag The drag due to interference effects between the fins and the body is given by equation (44)

$$C_{D(i)} = 2C_{f(f)}\left(1 + 2\frac{T_f}{l_m}\right)\frac{4n(A_{fp} - A_{fe})}{\pi d_f^2} \tag{44}$$

Viscous Friction As discussed in section 3.1 the viscous forces on the rocket are dependent on Reynolds number. The friction force coefficient is given by equation (45).

$$C_f = \begin{cases} \dfrac{1.328}{\sqrt{Re}} & \text{when } Re \leq Re_c \\ \\ \dfrac{0.074}{Re^{1/5}} - \dfrac{B}{Re} & \text{when } Re \geq Re_c \end{cases} \tag{45}$$

where B is given by equation (46).

$$B = Re_c\left(\frac{0.074}{Re^{1/5}} - \frac{1.328}{\sqrt{Re}}\right) \tag{46}$$

Reynolds number is given by

$$Re = \frac{\rho V L}{\mu} \tag{47}$$

where ρ is the atmospheric density, μ is the kinematic viscosity of air, V is the apparent velocity vector and L is the characteristic dimension. To calculate the viscous friction coefficient of the rocket body $C_{f(b)}$ the characteristic dimension is the total body length $L = l_{TR}$. To calculate the viscous friction coefficient of the rocket fins $C_{f(f)}$ the characteristic dimension is the fin mid chord $L = l_m$. Re_c is the critical Reynold number, which Mandell et al [1973] give as 5×10^5.

Total zero angle of attack drag The total zero angle of attack drag coefficient $C_{D(0)}$ can be modelled as sum of the individual drag coefficients given in equations (41) to (44)

$$C_{D(0)} = C_{D(fb)} + C_{D(b)} + C_{D(f)} + C_{D(i)} \tag{48}$$

Additional drag at angle of attack Mandell et al [1973] presents equations for estimating an additional component of drag, which can be added to the zero angle of attack drag ($C_{D(0)}$) in order to model the drag force at small angles of attack α. This method incorporates some coefficients that are derived from data which comes from wind tunnel experiments on rocket models.

As with the zero angle of attack drag force the component of *alpha drag* is subdivided into drag on the rocket body $C_{Db(\alpha)}$ and drag on the finset $C_{Df(\alpha)}$.

The coefficient of alpha drag on the rocket body is calculated using equation (49).

$$C_{Db(\alpha)} = 2\delta\alpha^2 + \frac{3.6\eta(1.36 l_{TR} - 0.55 l_n)}{\pi d_b}\alpha^3 \tag{49}$$

where α is the angle of attack of the rocket and δ and η are experimentally derived coefficients. These are obtained using experimental data from wind tunnel measurements. Figure 4 from Mandell et al [1973] shows δ and η

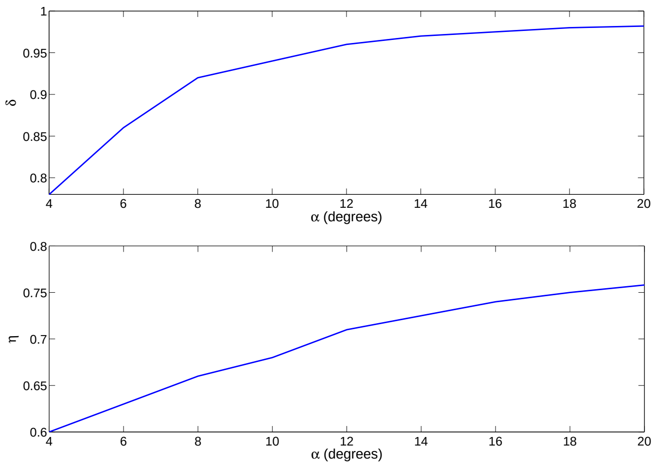

Figure 4: Graphs showing the relationship between α and the coefficients δ and η [Mandell et al, 1973].

The coefficient of alpha drag on the rocket's fins is calculated using

$$C_{Df(\alpha)} = \alpha^2 \left[1.2 \frac{A_{fp} 4}{\pi d_f^2} + 3.12(k_{fb} + k_{bf} - 1) \left(\frac{A_{fe} 4}{\pi d_f^2} \right) \right] \tag{50}$$

where A_{fp} is the total fin planform area and A_{fe} is the total fin exposed area. k_{fb} and k_{bf} are the fin-body interference coefficient and the body-fin interference coefficient given by equations (51) and (52).

$$k_{fb} = 0.8065 R_s^2 + 1.1553 R_s \tag{51}$$

$$k_{bf} = 0.1935 R_s^2 + 0.8174 R_s + 1 \tag{52}$$

where R_s is the fin section ratio, which is the ratio between the total span on the fins l_{TS} (figure 3) and the diameter of the body tube where the fins are mounted d_f, ($R_s = \frac{l_{TS}}{d_f}$).

The complete coefficient of drag on the rocket is obtained by adding the alpha drag coefficients to the zero angle of attack drag coefficient, hence

$$C_D = C_{D(0)} + C_{Db\alpha} + C_{Df\alpha} \tag{53}$$

Knowing estimates for the drag force coefficient C_D and the normal force coefficient C_N an estimate for the axial force coefficient C_A can be calculated using

$$C_A = \frac{C_D \cos \alpha - \frac{1}{2} C_N \sin(2\alpha)}{1 - \sin^2 \alpha} \tag{54}$$

This gives us two orthogonal force coefficients using the same convention (C_N and C_A).

3.2.3 Compressible Flow Correction

The methods described above for calculating C_N and C_A are only valid for incompressible flow. In this section we show how to extend the range of these aerodynamic coefficients into the compressible flow regime. The Prandtl-Glauert compressibility correction can be applied to estimate the variation in the aerodynamic force coefficient at trans-sonic speeds.

At subsonic speeds ($Ma < 1$) the corrected aerodynamic coefficient is given by equation (55) [Cramer, 2002]

$$C'_i = \frac{C_i}{\sqrt{1 - Ma^2}} \tag{55}$$

where C_i is the aerodynamic coefficient in the incompressible regime and Ma is the free stream Mach number.

At supersonic speeds ($Ma > 1$) the corrected aerodynamic coefficient is given by equation (56).

$$C'_i = \frac{C_i}{\sqrt{Ma^2 - 1}} \tag{56}$$

The problem with this estimation method is that C'_i goes to infinity as Ma goes to 1, which no longer fits experimental data. It is suggested in Ketchledge [1993] that in order to avoid this situation, the following equation is used for $0.8 < Ma < 1.1$.

$$C'_i = \frac{C_i}{\sqrt{1 - (0.8)^2}} \tag{57}$$

4 Summary

In this document it has been shown how to calculate a number of dynamic and aerodynamic parameters for rocket flight simulators. The parameters which have been dealt with are summarized in Table 1. As the table shows, many of the parameters can be functions of a variable in the rocket simulation such as time t, Reynolds number Re or angle of attack α. There are different ways that these dependencies can be dealt with, the parameters can be calculated on the fly for the simulator, or alternatively pre-calculated at discrete intervals and stored in a database to be accessed by the simulator and interpolated as required.

Symbol	Function of	Description
T	t	Thrust
M	t	Mass
X_{cm}	t	Distance of the centre of mass from the nose tip
I_{xx}	t	Moments of inertia about the rocket's yaw axis
I_{yy}	t	Moments of inertia about the rocket's pitch axis
I_{zz}	t	Moments of inertia about the rocket's roll axis
C_{da}	t	Thrust damping coefficient
C_N	$\mathrm{Re}, \alpha, \mathrm{Ma}$	Coefficient of normal force
C_R	$\mathrm{Re}, \alpha, \mathrm{Ma}$	Coefficient of aerodynamic roll torque
X_{cp}	α	Distance of the centre of pressure from the nose tip

Table 1: Parameters that are stored in the database for the rocket model.

The list of parameters that has been dealt with is not exhaustive. These parameters provide enough information for a simulator which assumes that the rocket is an axisymmetric rigid body. However for more complex simulations more parameters may be required, for example when simulating a non-axisymmetric rocket or modelling other phenomena that are neglected here such as thrust misalignment.

Nevertheless simulations using the axisymmetric assumption and the parameters addressed in this paper have been shown to be useful. Box et al [2009] used the methods laid out in this paper to provide inputs to a rocket flight simulator which was used to simulate the flight of a high powered rocket. The rocket was flown to an altitude of around 3.5 km before descending under parachute. Instrumentation on board the rocket recorded some data on the flight and the landing position was logged with a GPS receiver. Table 2 and Figure 5 from Box et al [2009] show some comparison between simulated and measured data. Various flight statistics are given in Table 2 and Figure 5 shows the flight path of the rocket ascent and parachute descent as predicted by the simulator. The ellipses in the figure mark the areas of 1σ and 2σ confidence in landing position. The diamond marker shows the recorded landing position of the actual rocket.

This is a single result and not a comprehensive test of the simulator or the methodology used to estimate the input parameters, but it is an encouraging result that suggests that future investigation into the validity of this methodology may be worthwhile.

	Simulated	Measured
Launch tower clearance velocity	40ms^{-1}	37ms^{-1}
Maximum velocity	372.5ms^{-1}	335ms^{-1}
Apogee altitude	3539m	3594m
Time to apogee	24.5s	24.5s
Total flight time	170s	182s
Landing position $[E, N]$	$[-135\text{m}, 936\text{m}]$	$[-71\text{m}, 1042\text{m}]$
Difference in landing positions	125m	

Table 2: Comparison between simulated and measured flight statistics.

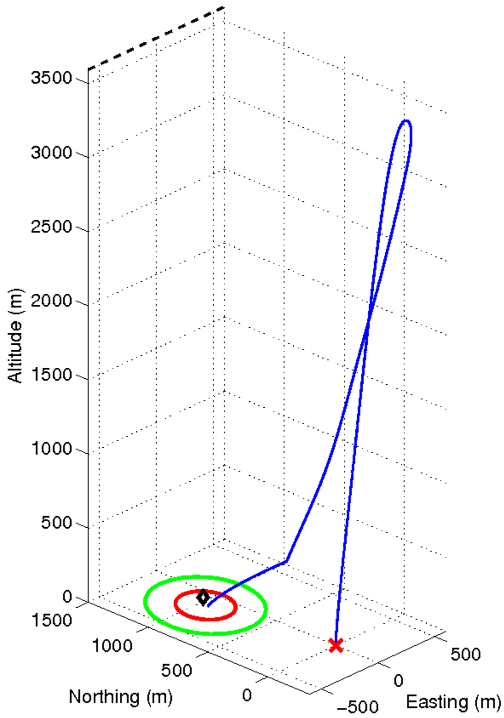

Figure 5: Mean simulated flight path with 1σ and 2σ landing probabilities. The measured landing position is marked by a diamond.

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